BRIEF COMMUNICATIONS

TEMPERATURE DISTRIBUTION AND HEAT TRANSFER IN A LAMINAR INCOMPRESSIBLE ANNULAR-CHANNEL FLOW WITH ENERGY DISSIPATION

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Expressions are obtained for the temperature distribution over the section, the heat flow through the channel wall, and the coefficient of heat transfer from the fluid to the wall for the case of a laminar flow in an infinite annular channel with constant specific heat flux at the outer wall of the channel and a thermally insulated inner wall taking energy dissipation into account.

This note presents the results of a solution of the differential equations of energy and motion, with allowance for energy dissipation, in the case of a developed laminar flow along an annular channel of infinite length. This makes it possible to calculate the temperature distribution over the cross section and the heat transfer from the fluid to the wall.

The following assumptions are made: 1) the flow is stationary and laminar, the fluid is incompressible and its physical properties are constant; 2) the flow is developed, i.e., hydrodynamically and thermally stabilized; 3) there are no internal heat sources in the flow; 4) the effect of body forces is negligibly small; 5) the specific heat flux at the outer wall is constant along the length of the channel, the inner wall of the channel is thermally insulated. With these assumptions the differential equations of motion and energy, in cylindrical coordinates, have the form [1]

$$\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial w_x}{\partial r}\right)=\frac{1}{\mu}\frac{\partial P}{\partial x}, \qquad (1)$$

$$\frac{\rho C_p}{\lambda}w_x\frac{\partial t}{\partial x}=\frac{1}{r}\frac{\partial}{\partial r}\left(r\frac{\partial t}{\partial r}\right)+\frac{\mu}{\lambda}\left(\frac{\partial w_x}{\partial r}\right)^2. \qquad (2)$$

From the heat balance condition for an element of the channel we have

$$\frac{\partial t}{\partial x}=\frac{2q_c r_2}{\rho C_p\bar{w}\,(r_2^2-r_1^2)}. \qquad (3)$$

Solving Eqs. (1) and (2) jointly, using (3), with the boundary conditions R = 1, $\partial\theta/\partial R = 0$—insulated wall, $R = R_2$, $\theta = 0$, we arrive at an expression for the fluid temperature distribution over the channel cross section

$$\frac{\theta}{A}=\frac{1}{16}\left(\frac{R_2}{R_2^2-1}-\frac{b}{A}\right)R^4-\left[\frac{R_2}{4(R_2^2-1)}-\frac{R_2}{4\ln R_2}-\frac{b}{A}\frac{R_2^2-1}{4\ln R_2}\right]R^2-\frac{R_2}{4\ln R_2}R^2\ln R-$$

$$-\frac{b}{A}\frac{(R_2^2-1)^2}{8\ln^2 R_2}\ln^2 R+\left[\frac{R_2}{4(R_2^2-1)}-\frac{R_2}{4\ln R_2}+\frac{b}{A}\left(\frac{1}{4}-\frac{R_2^2-1}{2\ln R_2}\right)\right]\ln R+$$

$$+\frac{3R_2^5+4R_2^3-4R_2\ln R_2-4R_2}{16(R_2^2-1)}-\frac{R_2^3}{4\ln R_2}+\left[\frac{3R_2^4+4R_2^2-4\ln R_2-6}{16}-\frac{R_2^2(R_2^2-1)}{4\ln R_2}\right]\frac{b}{A}, \qquad (4)$$

where

$$\theta=t-t_w(x),\quad A=\frac{\frac{\partial P}{\partial x}r_1^3 q_w}{2\mu\lambda\bar{w}},\quad b=\frac{r_1^4\left(\frac{\partial P}{\partial x}\right)^2}{4\mu\lambda},$$

and A and b are dimensionless groups of constants. Setting b = 0 in (4), we can obtain the temperature distribution for the case when the heat of friction is not taken into account.

Knowing the temperature distribution, we can easily find an expression for the specific heat flow through the outer wall:

$$q_c=\frac{r_1}{8}\frac{\partial P}{\partial x}\bar{w}\,\frac{R_2^3+\frac{R_2^4-1}{R_2\ln R_2}-\frac{1}{R_2}}{\frac{2\mu\bar{w}}{r_1^2\frac{\partial P}{\partial x}}-\frac{R_2^2+1}{4}+\frac{R_2^2-1}{4\ln R_2}}. \qquad (5)$$

The coefficient of heat transfer from the fluid to the wall is given by

$$\alpha=\frac{q_w}{\bar{\theta}}, \qquad (6)$$

where

$$\bar{\theta}=\frac{\int\limits_{R=1}^{R=R_2}2\pi R\,\theta(R)\,dR}{\pi(R_2^2-1)}. \qquad (7)$$

Using (4), we have

$$\frac{\bar{\theta}}{A}=\frac{R_2^3\ln R_2}{4(R_2^2-1)^2}+\frac{4R_2^5-13R_2^3-15R_2-4R_2\ln R_2}{48(R_2^2-1)}+\frac{9R_2-3R_2^3}{32\ln R_2}+$$

$$+\frac{b}{A}\left[\frac{R_2^2\ln R_2}{4(R_2^2-1)}+\frac{2R_2^4-7R_2^2-12\ln R_2-25}{48}+\right.$$
$$\left.+\frac{R_2^4+2R_2^3-3}{8\ln R_2}-\frac{(R_2^2-1)^2}{16\ln^2 R_2}\right]. \qquad (8)$$

Thus, Eqs. (4), (5), and (8) make it possible to calculate the heat transfer from a laminar flow of fluid with constant physical properties in an infinite annular duct with allowance for the heat of friction. These expressions were obtained for an adiabatically insulated inner wall and constant specific heat flux at the outer wall—along the length of the channel. It is assumed that the constant pressure gradient $\partial P/\partial x$ and the mean velocity in the section $\bar{w}$ are known experimentally.

NOTATION

w_x is the flow velocity; $\bar{w}$ is the mean velocity; x is the coordinate along the flow; r is the radius; r_1 and r_2 are the radii of the inside and outside channel surfaces, respectively; $R = r_1/r_2$ is the dimensionless radius; μ is the dynamic viscosity; P is the pressure; ρ is the fluid density; λ is the fluid thermal conductivity; c_p is the specific heat of the fluid; q_w is the specific heat flow through the wall; t is the fluid temperature; θ is the fluid temperature reckoned from the temperature of outside surface of the tube; α is the coefficient of heat transfer from the fluid to the wall.

REFERENCE

1. H. Gröber, S. Erk, and U. Grigull, Fundamentals of Heat Transfer [Russian translation], IL, 1958.

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